Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 7, Issue 2 (2014): 71-76

RELIABILITY ESTIMATION FOR MEAN UNDER NON-NORMAL POPULATION AND MEASUREMENT ERROR

Ketki Kulkarni[1] and J. R. Singh[2]
School of Studies in Statistics, Vikram University, Ujjain, India
E Mail: [1]ketki333@rediffmail.com; [2]jrsinghstat@gmail.com

Received May 09, 2014
Modified November 19, 2014
Accepted November 27, 2014

Abstract

Effect of non-normality and measurement error on $\tilde{R}$ (t) function has been studied. Numerical results are given to illustrate the mathematical findings.

Key Words: Edgeworth Series, Standardized Cumulants, Reliability Function, Non-Normal Population.

1. Introduction

The reliability function considered in the present study, $\tilde{R}$(t), is the probability function that a given system or device will operate successfully for at least t time units. In reliability estimation for mean when standard deviation σ is known, it generally assumed that the sampled population is normal and observations are error free. On the basis of this assumption the reliability function for the mean is calculated. But in practice, most of the basic industrial variables do not satisfy these assumptions and hence one may doubt the validity of the inferences drawn from the reliability functions. To get a satisfactory result it is advocated that in such cases (i) to increase the sample size or (ii) to transform the variables so that transformed variate may approximately follow normal distribution. This may not be always feasible in practice as (i) the sample size may be fixed on so many other considerations (ii) it may be difficult to find a suitable transformation and to apply the transformation effectively.

Among other important life distribution, one parameter gamma distribution is considered by Basu (1964) and he has obtained the UMVU estimator of component reliability. Baikunth Nath (1975) has extended the result given by Holla (1967) to the case of truncated gamma distribution. Proceeding in the lines of Sathe and Varde (1969) and Scheaffer (1976) has obtained UMVU estimator of mean time in service when the life time follows exponential and that of Laplace transformation in the case of gamma distribution. Assuming that X and Y are independent normal variable. Church and Harris (1977) have obtained the UMVU estimators of the reliability P [Y > X] in stress - strength model. Folks and *Chhikara* (1978) have found the estimator of reliability function using complete samples in the case of Inverse Gaussian distribution.

Rao-Blackwell and Lehman-Scheffe Theorem: Let T be sufficient for θ and let T_1 be an unbiased estimator of ψ(θ) such that Var (T_1| θ) is finite. Then E(T_1|T=t) = u(t) is independent of θ and

(i) E [u(T)| θ] = ψ(θ),

(ii) Var [u(T)| θ] ≤ Var [T_1| θ].

If T is complete and sufficient for θ, then any function u (T) is uniformly minimum variance unbiased estimator (UMVUE) of its expectation.

In this paper an attempt has been made to study reliability estimation for mean under non-normal population and measurement error. The reliability function is derived by considering the first four terms of an Edgeworth series as the probability density function of the non-normal populations.

2. Reliability Function for Non-Normal Distribution

The non-normal population considered here is supposed to be characterized by non-zero values of the standardized third and fourth cumulants. Since the effects of the higher - order term depending on $\lambda_5, \lambda_6, \lambda_3, \lambda_4, \lambda_3^2$ ... are assumed to be negligible, the population covered is only moderately non-normal. Too high values of λ_3 and λ_4 can also not be permitted as they will make f (x) negative at one or both tails and will give the subsidiary modes. The values of λ_3 and λ_4 considered should within Barton and Dennis (1952) limits, which means that for such values the population is positive definite and unimodal.

Let μ and σ^2 denote the mean and variance of the true quality characteristics *x* and let $\lambda_3 = \left(\sqrt{\beta_1}\right)$ and $\lambda_4 = (\beta_2 - 3)$ be the standardized third and fourth cumulants respectively. Assume all the higher order cumulants to be zero, so that, to the third approximation of the law of error, the frequency function of *x* is represented by the first four terms of an Edgeworth series.

$$f(x) = \phi\left(\frac{x-\mu}{\sigma}\right) - \frac{\lambda_3}{6}\phi^{(3)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_4}{24}\phi^{(4)}\left(\frac{x-\mu}{\sigma}\right) + \frac{\lambda_3^2}{72}\phi^{(6)}\left(\frac{x-\mu}{\sigma}\right). \quad (2.1)$$

The distribution of sample mean obtained from Gayen (1949) is

$$g(\bar{x}) = \phi\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) - \frac{\lambda_3}{6\sqrt{n}}\phi^{(3)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) + \frac{\lambda_4}{24n}\phi^{(4)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) + \frac{\lambda_3^2}{72n}\phi^{(6)}\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) \quad (2.2)$$

where $\phi^{(v)}(x)$ denotes the v^{th} derivative of ϕ (*x*). Let $\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\right) = \bar{\xi}$, then following Zacks and Even (1966) we obtained the reliability estimation as :

$$\tilde{R}(t) = 1 - \int_{-\infty}^{z} g(\bar{\xi})\, d\,\bar{\xi}$$

$$= 1 - \int_{-\infty}^{z} \left\{ \phi(\xi) - \frac{\lambda_3}{6\sqrt{n}} \phi^{(3)}(\xi) + \frac{\lambda_4}{24n} \phi^{(4)}(\xi) + \frac{\lambda_3^2}{72n} \phi^{(6)}(\xi) \right\} d\,\xi$$

$$= 1 - \left[\Phi\,(z) - \frac{\lambda_3}{6\sqrt{n}} \phi^{(2)}(z) + \frac{\lambda_4}{24n} \phi^{(3)}(z) + \frac{\lambda_3^2}{72n} \phi^{(5)}(z) \right]. \qquad (2.3)$$

We now examine the effect of the measurement error on the usual test criterion of single sampling plan described below :

Accept the lot if $\bar{x} + k\sigma \le U$,

And reject otherwise,

for a given set of values of the producer's risk α, consumer's risk β, Acceptable Quality Level (AQL) p_1 and Lot Tolerance Proportion Defective (LTPD) p_2, the values of n (size of sample) and k (acceptance number) are determined by the formulae

$$n = \left[\frac{(K_\alpha + K_\beta)}{(K_{p_1} - K_{p_2})} \right]^2, \qquad (2.4)$$

$$k = \left[\frac{(K_\alpha K_{p_2} + K_\beta K_{p_1})}{(K_\alpha + K_\beta)} \right] \qquad (2.5)$$

where K_{p_1}, K_{p_2} K_α and K_β are determined by the equation

$$\int_{k_\theta}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2} \right] dt = \theta, \qquad (2.6)$$

for different choices of fraction defective θ. If θ is the proportion defective in the lot, we know that

$$\frac{U - \mu}{\sigma} = K_\theta. \qquad (2.7)$$

$$\tilde{R}(t) = 1 - \left[\Phi\,(z') - \rho^3 \frac{\lambda_3}{6\sqrt{n}} \phi^{(2)}(z') + \rho^4 \frac{\lambda_4}{24n} \phi^{(3)}(z') + \rho^6 \frac{\lambda_3^2}{72n} \phi^{(5)}(z') \right], \qquad (2.8)$$

where

$$z' = \frac{t - \bar{x}}{\sqrt{\frac{\sigma_p^2}{\rho^2}\left(1 - \frac{1}{n}\right)}}.$$

3. Illustration and Conclusions

This paper has considered the problem of reliability estimation for mean under non-normal population and measurement error. In order to see how the normal theory reliability estimation is distorted in a situation of non-normality and measurement error, we consider a few specific values of λ_3 and λ_4, To discuss the problem a simple example, is cited below from Sinha and Kale (1979).

Example

15 items were put on test and the failure times (in hours) were :
13.4, 14.2, 28.8, 29.0, 29.8, 33.0, 37.8, 39.6, 43.4, 49.8, 54.8, 58.2, 67.4, 70.2, 91.2.

In the study of life-testing and reliability analysis one important approach has been to consider an underlying 'life' distribution and to find suitable estimates of the parameters of that distribution. For practical reasons a relevant problem would be to get an unbiased estimate of reliability. Rao-Blackwell and Lehmann - Scheffe therorems are used to derive the minimum variance unbiased estimates of reliability for a distribution that has proved useful in life testing.

(λ_3, λ_4) t	0	0	0
12	0.9505	0.9305	0.9452
21	0.8829	0.8554	0.8749
30	0.7642	0.7389	0.758
39	0.6025	0.5909	0.5987
48	0.4208	0.4286	0.4247
57	0.2515	0.2776	0.2579
66	0.1293	0.1563	0.1357

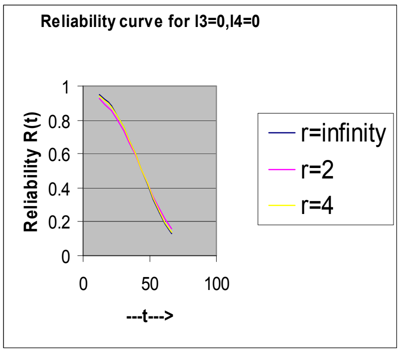

Table 1: Reliability Estimation for Measurement Error and Non-Normal Population for r = ∞.

(λ_3, λ_4) t	(0.5,2)	(0.5,2.0)	(0.5,2.0)
12	0.9471	0.9285	0.9422
21	0.883	0.856	0.8752
30	0.7704	0.743	0.7632
39	0.6115	0.5971	0.6069
48	0.4138	0.4232	0.4181
57	0.2502	0.2755	0.2563
66	0.1327	0.1575	0.1384

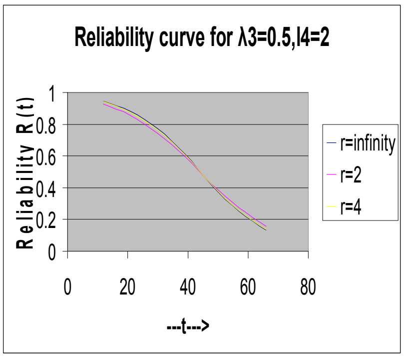

Table 2: Reliability Estimation for Measurement Error and Non-Normal Population for r = 2.

(λ_3, λ_4) t	(0.5, 0.5)	(0.5, 0.5)	(0.5, 0.5)
12	0.9469	0.9282	0.942
21	0.8815	0.8551	0.8738
30	0.7679	0.7419	0.7613
39	0.6103	0.5966	0.6059
48	0.4128	0.4228	0.4174
57	0.248	0.2744	0.2544
66	0.131	0.1566	0.1369

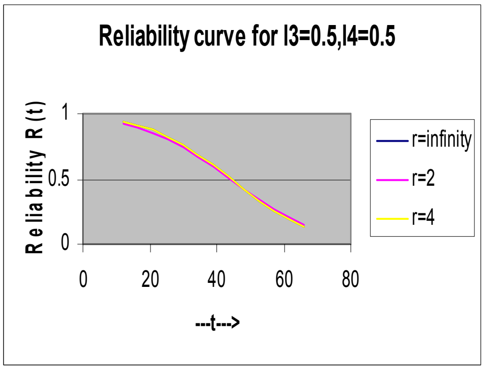

Table 3: Reliability Estimation for Measurement Error and Non-Normal Population for r = 4.

(λ_3, λ_4) t	(-0.5, +0.5)	(-0.5, +0.5)	(-0.5, +0.5)
12	0.9546	0.9331	0.9489
21	0.885	0.8561	0.8766
30	0.761	0.7362	0.7551
39	0.5948	0.5853	0.5919
48	0.4289	0.4345	0.4322
57	0.2555	0.281	0.2618
66	0.1283	0.1564	0.1351

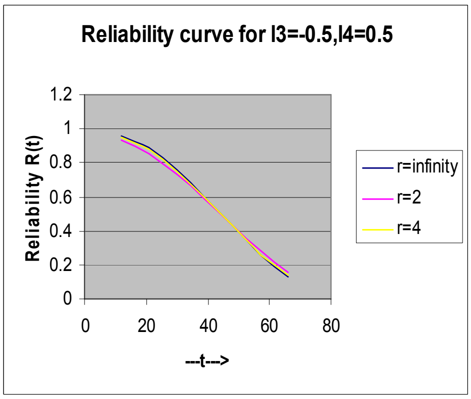

Table 4: Reliability Estimation for Measurement Error and Non-Normal Population for r = 6.

The values of $\tilde{R}$ (t) are given in Table1 to Table 4 for different values of λ_3, λ_4 and r (size of the measurement error). The reliability curves is drawn in Figure 1 to Figure 4 for different values of r. Visual comparison shows that $\tilde{R}$ (t) is not much for normal and non-normal population. The effect of kurtosis is negligible. As failure time increases the $\tilde{R}$ (t) decreases. As compare to non-normality the measurement error is quite serious on the reliability function. To a lepto-kurtic and positive skewness ($\lambda_3 >$ 0and $\lambda_4 > 0$), when normal-theory reliability function is compared an overall improvement is likely to result and for the values of λ_4 of order 2 it will be really a marked improvement. Positive skewness tends to improve the reliability function only in a limited range of failure rate. However, the presence of both skewness and kurtosis would affect a $\tilde{R}$ (t) depends on the magnitudes of λ_3 and λ_4 in a particular case and their effects being additive. True values and random components are additive in nature.

References

1. Baikunth Nath, G. (1975). Unbiased estimates of reliability for the truncated gamma distribution, Scandinavian Actuarial Journal, Volume 1975(3), p. 181-186.
2. Barton, D.E. and Dennis, K.E. (1952). The conditions under which Gram-Charlier and Edgeworth curves are positive definite and unimodal, Biometrika, 39, p. 425-427.
3. Basu, A.P. (1964). Estimates of reliability for some distributions useful in life testing, Technometrics, 6, p. 215-219.
4. Church, J.D. and Harris, B. (1977). The estimation of reliability from stress - strength relationship, Technometrics 12, p. 49-54.
5. Folks, J.L. and Chhikara, R.S. (1978). The inverse Gaussian distribution and its statistical application - a review, Journal of the Royal Statistical Society B 40, p. 263-275.
6. Gayen, A.K. (1949). The distribution of student's t in random samples of any size dream from non-normal universes, Biometrika, 37, p. 236-255.
7. Holla, M.S. (1967). Reliability estimation of the truncated exponential model, Technometries, 9, p. 332-335.
8. Sathe, Y.S. and Varde, S.D. (1969). Minimum variance unbiased estimators of reliability distribution, Technometrics, 11, p. 609-612.
9. Scheaffer, R.L. (1976). On the computation of certain minimum variance unbiased estimators, Technometrics, 18, p. 497-499.
10. Sinha, S.K. and Kale, B.K. (1979). Life Testing and Reliability Estimation, Wiley Eastern Limited, New Delhi.
11. Zacks, S. and Even, M. (1966). The efficiencies in small samples of the maximum likelihood and best unbiased estimators of reliability function, JASA, 61, p. 1033-1051.